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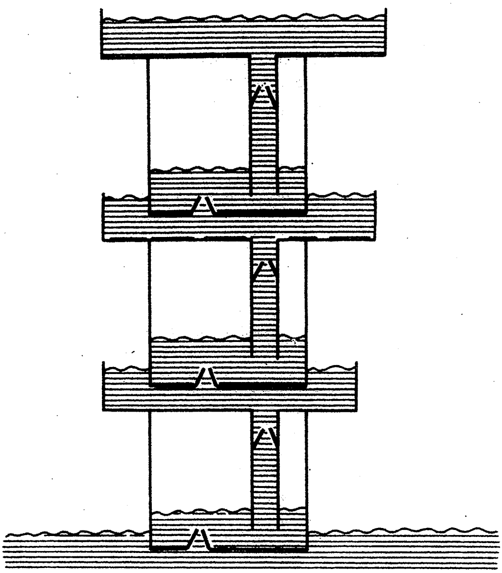

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Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting of traffic lights, that do not involve mathematics. *Function* contains articles describing some of these uses of mathematics. It also has articles, for entertainment and instruction, about mathematics and its history. Each issue contains problems and solutions are invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

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If we were to give a theme to this issue of *Function*, perhaps it would be the interaction of mathematics with the everyday world. Over 4000 years ago a practical problem was committed to papyrus and its answer found. In later days, the matter of the volume of wine casks became important. Now, a new pump. Enjoy also Dr Bowers' article on ... - well read it for yourself.

THE FRONT COVER

Dr Carl Moppert, in his third appearance in *Function*, describes another of his inventions, a pump. Our cover shows three such pumps in series, one above the other, so that each pumps fluid into the one above it. This allows, in principle, water to be pumped to great heights. Dr Moppert suggests that some such mechanism may draw the sap up to the tops of trees.

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THE M-PUMP

Carl Moppert, Monash University

The M-pump (why M?) is of course one of the greatest inventions of this century. It solves all the problems of irrigation. It is an inexhaustible source of energy and absolutely non-polluting. In fact, it disposes of otherwise polluting empty beer cans.

Apart from its practical application, the principle of the M-pump solves a problem which eluded scientists for hundreds of years: how does the sap get up a tree?

This being the first published account of the M-pump, the present issue of *Function* will have a high value for collectors in the future.

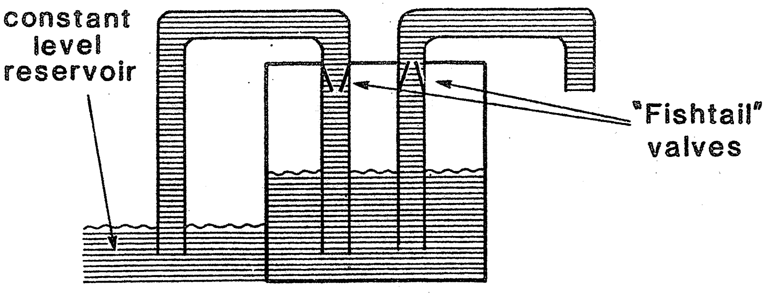

Figure 1: The principle of the M-pump

A tin is partly filled with water. Two pipes are glued in the top of the tin, both reaching down to the water inside. One of the pipes (the left one in the figure) reaches into a reservoir of water. The other pipe bends over: out of this end the water flows. In each pipe is a valve. The valve in the left pipe opens up for water running down, that in the right pipe for water running up. Either valve closes if the water wants to run in the opposite direction.

The working agent of the pump is the air above the water in the tin. If the temperature increases, this air wants to expand and the pressure in the tin increases. Some of the water in the tin is driven out through the right-hand pipe (like coffee in a percolator). The water cannot escape through the left-hand pipe because of the valve. If now the temperature decreases, the air in the tin wants to contract. The pressure decreases and water is sucked in through the left-hand pipe. It cannot be sucked in through the right-hand pipe because of the valve there.

Whenever the temperature increases, water will come out of the right-hand pipe and whenever the temperature decreases, water will be sucked in through the left-hand pipe.

In our discussion, we shall assume that the barometric pressure remains constant and that the temperature fluctuates. It is true of course that our pump also acts if the temperature remains constant and the barometric pressure fluctuates, but we will not go into this.

The pump is "driven" by changes of temperature. The one I have built is kept on the veranda of my house; the changes from day temperature to night temperature are sufficient to drive it. The best pumping action is observed on a summer day with some clouds in the sky: the changes of temperature have a very marked effect.

The Theory

All we need to know is the so-called *gas law*: for any quantity of gas (air, oxygen, hydrogen or whatever) the expression pV/T is constant, where p is the pressure, V the volume and T the absolute temperature (degrees Celsius + 273°). Pressure and volume can be measured in any old units, as long as we stick to them.

For present purposes, I shall measure p in cm. of water, V in cm^3. Atmospheric pressure is then about 1000 cm (10 metres of water correspond to 750 mm of mercury).

As the air in the tin never changes - only water is coming in or going out - for this air the expression pV/T always has some value k say:

$$pV/T = k \, . \tag{1}$$

We look at the situation when the absolute temperature of the air in the tin takes its maximum value T_{max}. Let then p_{max} and V_{max} denote the corresponding values of p and V. Furthermore, let B denote the outside barometric pressure (in cm. of water).

At T_{max}, the water just stops coming out of the right-hand pipe. At the end of this pipe, the pressure is then B. As this pipe is full of water, we have this same pressure in the pipe *within* the tin at the point which is on the level of the end of the pipe (see Figure 2).

In Figure 2, the bottom of the tin is at the level of the (essentially infinite) outside reservoir of water and the opening of the right-hand pipe is at the same level as the top of

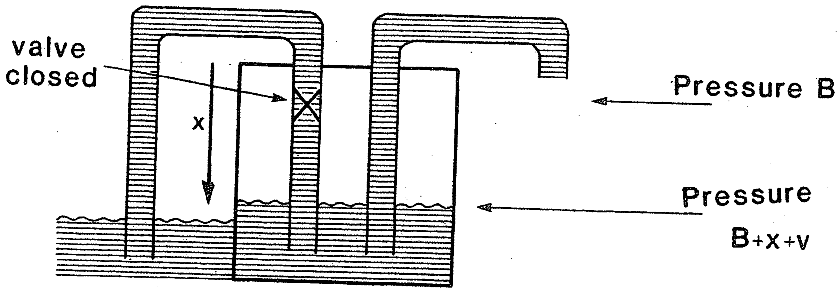

Figure 2. Pressure distribution at the opening of the R.H.valve

the tin. The water in the tin is x cm. below the top of the tin. The air in the tin then occupies the volume

$$V_{max} = xQ \ , \tag{2}$$

where Q is the cross-section of the tin (less the cross-sections of the pipes) in cm^2. The pressure p_{max} in the tin is

$$p_{max} = B + x + v \tag{3}$$

in cm of water, where v is the pressure (in the same units) required to open either valve.

Equations (1), (2) and (3) then give

$$xQ(B + x + v) = k\ T_{max}. \tag{4}$$

At this stage, we need not worry about the left-hand pipe as the valve in it is closed, while water comes out of the right-hand valve.

At the minimum temperature T_{min}, we make the corresponding calculations. The air in the tin has then contracted and therefore the water level inside the tin is higher. The valve in the right-hand pipe is closed and that in the left-hand pipe is open.

From Figure 3, we see that the air now occupies the volume V_{min}, namely

$$V_{min} = yQ \ , \tag{5}$$

and that its pressure is

$$p_{min} = B - h + y - v \ . \tag{6}$$

(The term $-v$ indicates that the valve in the left-hand pipe has to be kept open.)

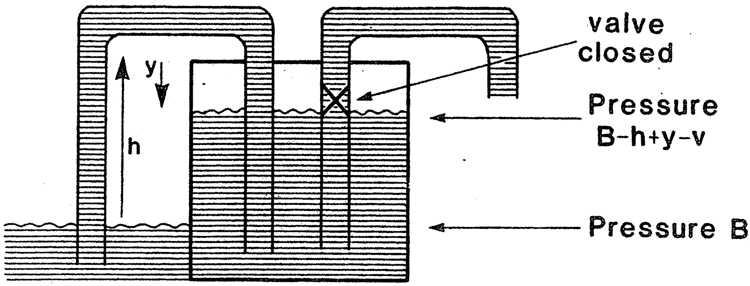

Figure 3. Pressure distribution at the opening of the L.H.valve.

From (1), (5) and (6), we find then

$$yQ(B - h + y - v) = k\, T_{min}\,. \tag{7}$$

The constant k in (4) and (7) is the same as the air in the tin has not changed.

From (4) and (7) we eliminate both Q and k at one stroke:

$$\frac{x(B + x + v)}{T_{max}} = \frac{y(B - h + y - v)}{T_{min}}\,. \tag{8}$$

In Equation (8), the barometric pressure B and the pressure v needed for opening the valves are given. We also take the temperatures T_{min} and T_{max} as given. Equation (8) is then the equation of a hyperbola in x and y; its axes are parallel to the coordinate axes.

In order to have a positive pumping action we must have

$$y < x\,. \tag{9}$$

In fact the volume of water pumped is $(x - y)Q$. Accordingly, we have to consider (8) and (9) together.

As always with such questions, we look for the solutions with $y = x$ in Equation (8). There are two solutions, namely

$$x = y = 0$$

and

$$x = y = -B + v\,\frac{T_{max} + T_{min}}{T_{max} - T_{min}} + h\,\frac{T_{max}}{T_{max} - T_{min}}\,.$$

The second solution is negative as B is large compared with h and v. For positive values of x and y we thus have either $y < x$ always or $y > x$ always. The first case applies if

$$y'(0) < 1\,, \tag{10}$$

where y is considered as a function of x in Equation (8).

From (8) we find by differentiating with respect to x :

$$\frac{B + 2x + v}{T_{max}} = \frac{B + 2y - h - v}{T_{min}}\, y' \tag{11}$$

and this gives for $x = y = 0$

$$y'(0) = \frac{T_{min}}{T_{max}} \cdot \frac{B + v}{B - h - v}\;. \tag{12}$$

Taking (10) and (12) together then gives the condition

$$h\,\frac{T_{max}}{T_{max} - T_{min}} + v\,\frac{T_{max} + T_{min}}{T_{max} - T_{min}} < B\,. \tag{13}$$

If (13) is not satisfied, the pump does not work at all. It follows that h and v must be kept small. For T_{min} = 283°abs = 10°C, T_{max} = 308° abs = 35°C, B = 1000 cm, Condition (13) gives

$$12{\cdot}3h + 23{\cdot}6v < 1000.$$

We are lucky if we find valves with v = 10 cm, and even then we must have h < 62·1 cm. Of course, h must be kept considerably lower to allow an appreciable pumping action.

For $x = h = 20$ and T_{min}, T_{max} and B as before, we find from (8) that y = 19·16 cm. Accordingly 0·84Q cm^3 of water are then lifted by h(= 20)cm. This agrees well with my experiments. For $h = x = 10$, we have y = 9·48 cm, i.e. 0·52Q cm^3 of water are lifted by 10 cm.

If $x = h$, the greatest volume of water will be lifted as then the greatest volume of air is expanding. In order to pump water to any given height we stack many pumps, each of height h, on top of each other. We have then $n = 100/h$ pumps per metre and if each of these "cells" transports $Q(h - y)$ cm^3 of water, the stack of n cells pumps $100Q(h - y)/h$ cm^3 of water to the height of one metre. The efficiency of the stack of cells is thus measured by the quantity

$$E = \text{Efficiency} = \frac{h - y}{h}\,, \tag{14}$$

where y is determined by (8), i.e.

$$y(B - h + y - v) = \frac{T_{min}}{T_{max}}\, h\,(B + h + v)\,. \tag{15}$$

In our example, we have:

$$h = 10 \text{ cm} \qquad E = 0{\cdot}052$$
$$h = 20 \text{ cm} \qquad E = 0{\cdot}042\ .$$

It is not difficult to show that E always decreases with increasing h and becomes zero when h reaches the maximum value, given by (13). The maximum value E_{max} is then

$$E_{max} = \lim_{h \to 0} \frac{h - y}{h} \tag{16}$$

and this can be found to be

$$E_{max} = 1 - \frac{T_{min}}{T_{max}} \cdot \frac{B}{B - v} \quad . \tag{17}$$

In our example with $B = 1000$ cm, $v = 10$ cm, $T_{min} = 238°$abs, $T_{max} = 305°$ abs, we find $E_{max} = 0{\cdot}063$. For $v = 0$, we would have $E_{max} = 0{\cdot}072$.

I started thinking about this pump by trying to find an explanation of how the sap gets up a tall tree. As far as I know, the biologists don't have a satisfactory answer. My pump would give a possible mechanism, although I am told there are no valves to be found in the cells of a tree. Perhaps, however, if one looks for valves, one can find them. A point in favour of my theory is the fact that, given h and v, Equation (13) gives a minimum value of B. Accordingly, given h and v, the pump works only for sufficiently high barometric pressure. This would then explain why trees grow only up to a certain height above sea level.

Engineers have told me that my pump is not new. It is true that in 1698 Thomas Savery patented a similar pump. There are however essential differences: Savery used the expansion of steam and his valves were not automatic.

I have built several models of my pump and they keep working without any attention. As valves I use those from blood-transfusion apparatus. My pumps do not move mountains: a pump with a tin of volume 1 litre lifts about 100 cm^3 of water by 20 cm over 24 hours.

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MATHEMATICS AND DEDUCTIVE REASONING

In the great inquiries of the moral and social sciences ... mathematics (I always mean applied mathematics) affords the only sufficient type of deductive art. Up to this time, I may venture to say that no one ever knew what deduction is, as a means of investigating the laws of nature, who had not learned it from mathematics, nor can any one hope to understand it thoroughly who has not, at some time in his life, known enough of mathematics to be familiar with the instrument at work.

J.S. Mill

WINE CASKS AND CALCULUS†

Malcolm J. Cameron,

Burwood, Victoria

The Grape Harvest of 1612

The year 1612 was long remembered for its excellent fruity wine. Yet the grape harvest of that year produced more than wine. In fact the mathematics developed to cope with the problems of supplying sufficient wine casks to hold the fruits of the vine is the greatest mathematical achievement.

The year 1612 was indeed an outstanding vintage. Here's how it came about.

Good Fortune and its Problems

Last year's casks will normally suffice for this year's wine. If the harvest is poor there are casks to spare. But if the people are blessed with a really bumper harvest they must busy themselves constructing new wine casks.

And indeed they were busy in 1612.

So much so that the management decided to call in a consultant, a mathematical consultant. The obvious choice was Johannes Kepler, previously 'Imperial Mathematician' to Emperor Rudolph II of Bohemia. A bit of a mystic, yes, but evidently suitable for casting horoscopes, this being the reason for his previous employment. And who cared that Johannes spent his spare time reading old Greek texts propounding a new mystical cosmology and stargazing at the local observatory?

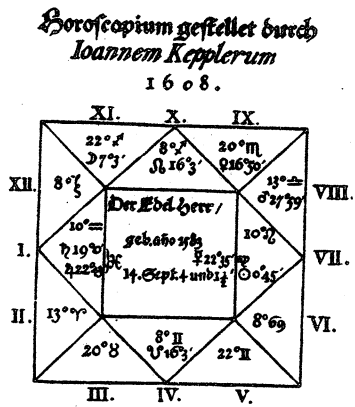

Figure 1. A horoscope cast by Johannes Kepler.

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†Reprinted with permission from the *Wine and Spirit Buying Guide*, Jan. 1983. See also the author's *Heritage Mathematics* (Melbourne: Hargreen, 1983), Chapter 10.

The Problem

As the harvest continued to ripen it was becoming painfully obvious that there was to be a severe shortage of wine casks. A tragedy.

More so because no-one knew with any accuracy how much wine the existing barrels would hold or which was the most cost-efficient barrel to construct. And there were many types to choose from and many being frantically constructed.

So Kepler was given two problems for his meditation:

- to improve upon the crude methods then used for estimating the volume of the casks.

- to determine the best method of barrel construction.

Of course a modern consultant would jump straight into the problem using calculus. This mathematics is just made for finding areas, volumes and maximums or minimums.

But calculus had not been invented then. So Kepler invented it. Or at least was one of a series of brilliant men responsible for its invention in a flowering of creative science unequalled since the great days of Greece.

The Old Method - Gauging

The obvious starting point was the booklet by Johann Frey printed in Nuremberg in 1531. This gave the standard method for calculating the capacity of a cask - called gauging.† But Kepler was unimpressed, as were the wine merchants.

Kepler next tried an old Greek text by Archimedes which had reached Western Europe in about 1450 via some Arab mathematicians. This he found to be a rigorous but sterile method. Once a formula is known it gives an elegant way of proving it, but it is of no use in discovering the initial result. (While Kepler was fortunate to have this Greek manuscript at all, it is unfortunate that he did not have another one. For it turned out that Archimedes had a second method - the closest the Greeks got to calculus. But this was not to be rediscovered until 1906 as a tenth century copy overwritten in the thirteenth century by a religious text - a sensational discovery at the time but another story.)

The New Method - Calculus

In searching for an alternative method, Kepler was one of the first to develop the idea of infinitesimal quantities. With these small quantities a point generates a line by motion, the line generates a surface as threads produce cloth, and a surface generates a solid, like pages of a book.

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†"Gauging" refers to the measurement of the volume of wine in a cask by measuring the wetted length on a standard dip-stick. Any individual cask can be calibrated to make the method very accurate, but the conversion of length to volume will be different for each cask unless the casks all have the same shape and size.

Figure 2. Calculation of the capacity of a wine cask; title page of a booklet by Johann Frey, Nuremberg 1531.

This was primitive calculus, complete with several self-contradictions which did not trouble him. He relied on divine inspiration writing that "nature teaches geometry by instinct alone..." Moreover his formulas for the volumes for wine casks are correct. They are recorded in his academic treatise 'Stereometria doliorum' or 'Volume Measurement of Barrels' (1615).

The Austrian Wine Cask

Not content with accurately determining the volume of wine in a particular barrel, Kepler looked at the best proportions for barrel construction using the same quantity of wood. This is the problem of optimum shape. When he listed the volume of wine for various barrel dimensions he noticed an interesting fact - as the maximum volume was approached the change in volume for a given change in the dimensions became smaller. (Figure 3.)

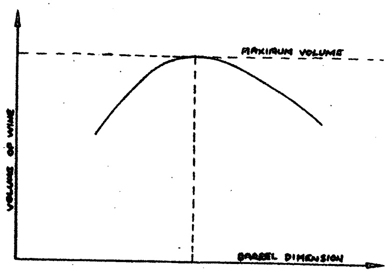


Figure 3. 'When he listed the volume of wine for various barrel dimensions he noticed an interesting fact...'

This was the basis, at least, for the later method of calculating maxima and minima of functions. Namely $\frac{dV}{dx} = 0$ when V, the volume, is maximum or minimum.

However it had to wait until Descartes' co-ordinate geometry of 1637 and the full development of calculus later.

All in all, did the wine merchants get value for their consulting fees? Yes, they got both accurate volumes for their barrels, and Kepler showed them that the Austrian-type of wine barrel approximated the optimum closely. Just as the Emperor Rudolph had got good horoscopes before his political problems had necessitated economy cuts in the royal household.

The Consultant

Kepler's 'Stereometria doliorum' will find applause among those who say that all science should be for practical application. Nevertheless it was far from his greatest triumph. Then again it easily surpassed his first dismal attempt at greatness.

His first attempt at greatness was related to the newly found 'Elements of Euclid' which had come from Ancient Greece via the Arabs, as with Archimedes' work. Now the last proposition of the last book of the 'Elements' showed that there can only be 5 regular solids (Figure 4). These are the tetrahedron (4 faces), the cube

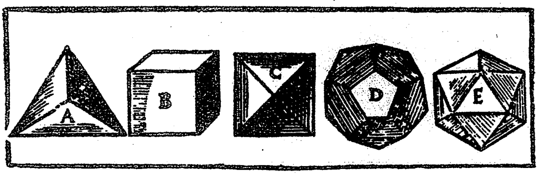


Figure 4. The five regular geometrical solids from 'Instrumentorum Mechanicorum' by Levinus Halsius, 1604.

(6 faces), the octahedron (8 faces). The dodecahedron (12 faces), and the icosahedron (20 faces).

Surely the last proposition of the venerable 'Elements of Euclid' must have mystical significance! So much so, that Kepler built up a cosmology about these 5 regular solids and the 5 intervals between the 6 known planets believing they must have been the creator's key to the structure of the heavens. (Figure 5.)

Needless to say Kepler's model was not successful then or after the discoveries of Neptune, Uranus and Pluto. Not that Kepler would admit it as he remained determined to prove the model to the end of his life.

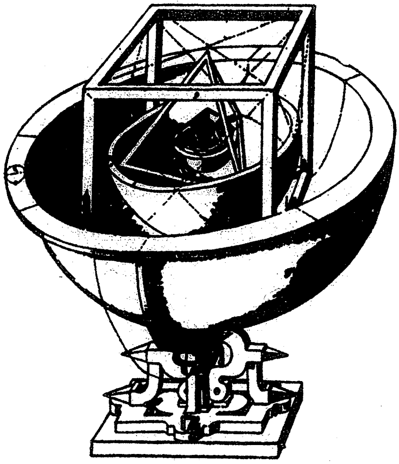
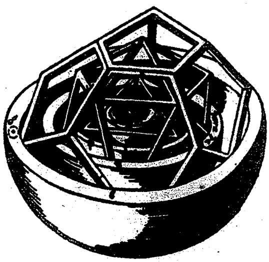


Figure 5(a). Kepler's model of the universe; the outermost spheres are Jupiter and Saturn. Illustration in Kepler's 'Mysterium Cosmographicum' 1596.



(b) Detail showing the spheres of Mars, Earth, Venus and Mercury with the Sun at the centre.

Music of the Celestial Spheres

Much of the ancient Greek texts concerned the philosophy of the Pythagoreans. They believed that 'All is number' as seen in the harmonious musical tones emitted by stretched strings whose lengths are in the ratio of whole numbers.

With bold imagination the Pythagoreans extrapolated that the planets emitted harmonious tones in their orbits, just as a weight whirled on the end of a string makes a sound. For the planets this is 'the harmony of the celestial spheres', a phenomenon said to be heard by veteran wine drinkers.

Now 'Imperial Mathematician' Kepler had come into possession of a set of meticulous observations of Mars upon the death of his predecessor Tycho Brahe. Believing mystically that there was order and beauty in the universe his persistance triumphantly lead to his three laws:

(a) The planets travel around the sun in elliptical orbits, one focus of the ellipse being occupied by the sun.

(b) The speed of the planet in its orbit is such that a line drawn from the planet to the sun sweeps out equal areas in equal times.

(c) The square of the time which a planet takes to go around the sun is proportional to the cube of the distance from the sun.

All very well of course. It altered the intellectual life of Europe, allowed Newton to formulate his law of gravitation, and allowed Halley to predict the return of his comet - due, incidentally, to visit us again in February 1986.

Yet by 1611 Kepler was unemployed. Money was tight and he struggled for existence. He suffered religious persecution and ill health while trying to publish his books, collect unpaid salary, and search for positions. Thus he died in 1630, leaving a verse for friends to place on his tombstone:

Once I measured the skies,
Now I measure the earth's shadow;
Of heavenly birth was the measuring mind,
In the shadows remains only the body.

For Kepler, was it worth it? No! He *had* discovered the music of the planets, in fact a concert. Yet science has ignored this, choosing only to retain the purely mathematical laws.

In his memory, this is the music of celestial spheres from Kepler's book 'The Harmony of the World',1619, where the tone depended on the variable speed of the planet in its orbit.

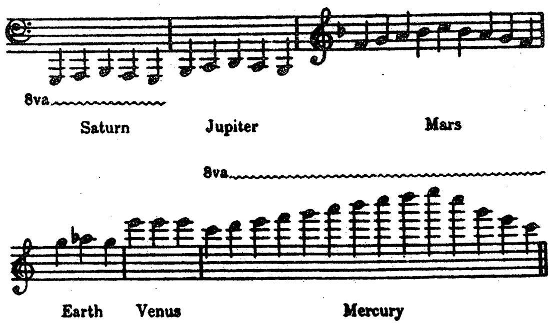


Figure 6. The music of the celestial spheres.

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A PYTHAGOREAN SNIPPET

Is there a Pythagorean triangle with a square hypotenuse and legs that sum to a square?

Yes, however the smallest answer is 4,565,486,027,761, 1,061,652,293,520 and 4,687,298,610,289. This was solved by Fermat in 1643.

Submitted by Garnet J. Greenbury.

WHY ARE MATHEMATICIANS ECCENTRIC?†

John F. Bowers, University of Leeds

The eccentricities of the greatest mathematicians are notorious. For example, when Archimedes's bath overflowed he did not behave like any ordinary citizen by cussing and then asking his slave to mop up the mess. Instead, he leapt out of the bath and ran shouting down the street. Further, it is well known that Newton saw an apple fall in his orchard. However, he did not indulge in trivial reflections as to whether he had remembered to spray the tree or whether the apple was fit to eat. Instead he had thoughts of great gravity. Gauss was, perhaps, the greatest of all mathematicians and, warned by tales about his predecessors, he carefully avoided all occasions of eccentricity such as public baths and old orchards, but his care only led him to become excessively normal. Could it be that these eccentricities were caused by greatness and not by mathematics? As can be deduced from the graph in Figure 1,

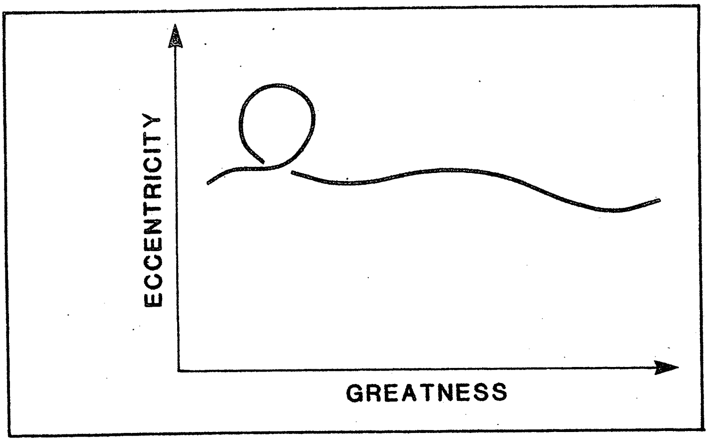

Figure 1.

†Reprinted, with slight abridgements, courtesy of Dr Bowers and *New Scientist*. The original appeared 22 December, 1983.

the relationship between greatness and eccentricity is not simple, but, to some extent, eccentricity declines with increasing greatness (just as the virulence of complaints decreases with increasing inconvenience, in accordance with the Inverse Swear Law). Clearly, we must investigate just how serious is the threat to mental health which is induced by mathematics.

In order to determine the effect that mathematics has on mathematicians we will need to conduct a sociological investigation. Also, before we answer our main question, we must first decide what we mean by "mathematician". This term is frequently used in very restrictive senses, but we need to use it in a way that is consistent with the usage of previous centuries. Accordingly, let us agree that a *mathematician* is somebody whose work consists wholly of creating, using or teaching mathematics, together with the administrative work that such activities entail. Therefore, among the mathematicians, we include all theoretical physicists, some philosophers and some engineers. Also, much of our study will apply partially to many other scientists and engineers in proportion to the time they devote to mathematics. Our definition is not exact, so the reader can be certain that this discussion will be entirely free of the pedantic humour that mathematicians commonly manufacture out of the misuse of precise definitions. Of course, rigorous definition is the basis of the quintessential mathematical form of humour: the application of an idea to itself. We clearly cannot employ this form of humour here because no mathematician can properly define a mathematician.

We can investigate a set of people by means of their *archetype*, the mental picture that others have of them. For the man in the street, the word "mathematician" immediately conjures up a picture of a short, bald, bearded and bespectacled man who is absent-minded and far from rich. However, this is in direct opposition to the following dictum, which may be verified at any large mathematical meeting:

Mathematicians are strictly irregular.

In order to test the validity of the archetype, a certain set of 56 mathematicians was analysed and it was found that:

2 were unusually tall,
2 were unusually short,
5 were bald,
4 had beards,
32 wore spectacles other than reading glasses,
52 were men.

This suggests that there is some truth in the statement that mathematicians are often men who wear glasses. However, the idea that mathematicians are absent-minded is absolutely wrong. There is a conclusive proof that shows that they are not, but unfortunately it cannot be given here because it seems to have been mislaid. On the other hand, if mathematicians are not necessarily poor, the study of mathematics is not a quick route to great wealth, although Newton certainly made a lot of money when he was Master of the Royal Mint.

The puzzling fact is the small proportion of mathematicians who are women, even though women have shown a talent for the subject for so long and there is no obvious physical or social barrier against them. Lists of mathematicians at one college were studied and it was found that women formed 2/7 of the honours undergraduates but 1/13 of the postgraduates and the staff. (There is no significant difference between those two classes.) If these figures are at all typical, it would seem that factors outside mathematics cause many talented women to forego postgraduate work. However, the percentage (30 per cent) of women undergraduates is well under the percentage of girls in the schools (about 45 per cent), so there may be some loss of girl mathematical scholars before higher education is reached. Perhaps the senior girls in a sexually segregated school do not feel the demands of economic life in the way that causes boys of the same age to accelerate their working sharply. On the other hand, a universal change to mixed schooling would deprive the upper years of many boys who, earlier, abandon all studies due to demoralising comparisons with the girls of the same age, who are physically and mentally more advanced. Fortunately, there is an ideal solution: all girls should go to mixed schools and all boys should go to segregated schools.

The reason for the general failure of this achetype is that it is archetypical in being a conscious creation, a deliberate physical representation of one who is a "desiccated calculating machine". But this last phrase was actually invented to describe Hugh Gaitskell, who was an economist. Unlike him, mathematicians rarely rise to positions of power, though one of them, Eamon de Valera, became President of Republic of Ireland. To discover an archetype that is less deliberately fabricated it is necessary to find the image of a mathematician in the public's subconscious mind. In order to find the *veritas* which is most likely *in vino*, the author has dutifully attended many social gatherings and has found the following encounter to be typical.

"Tell me," the managing director asks complacently on the other side of a large whisky, "what you do for a living."

"I am a mathematician."

The director's smug expression is instantly replaced by a horrified look and he makes a barely successful attempt to stop himself from stepping backwards as he replies,

"Oh ... I was never any good at that ...".

This reaction reveals a view of mathematicians as people of such superior intelligence that they are dangerous to know, and probably mad and bad as well. In order to avoid the social leprosy that is engendered by this attitude, mathematicians have developed a number of defensive techniques. The most inspired is John Williamson's reply:

"I am an analyst."

This is reported to produce reactions which we would not expect, even in our wildest dreams. A more widely available reply is for the mathematician to admit to being a teacher of tautology.

This normally produces a neutral reaction suffused with intellectual confusion, which the mathematician can then exploit deviously. (An occasional alternative reaction in the discovered presence of a mathematician is the reply: "I was always good at arithmetic at school", which depicts mathematicians as puerile purveyors of numerological trivia.)

Another method for discovering the common features of a set of people is to study their literary depiction. That is, we find out what the great writers think about them, because presumably, these writers have sources of knowledge that are not available to lesser men. For mathematicians this method gives poor results. Chaucer saw no mathematicians among the Canterbury pilgrims and, in *The Divine Comedy*, Dante listed none among the residents of Hell or Purgatory, consequently they must all be in Heaven. Shakespeare has no mathematicians in his plays and the plays of Chekov and Ibsen contain doctors, philosophers, engineers and literary scholars, but no mathematicians. No mathematicians intruded in the gentle life as portrayed by Jane Austen, Henry James or George Eliot. The novels of Graham Greene contain salesmen and journalists and those of Charles Dickens contain thieves and murderers, but no mathematicians. Worse, when Faust, in Goethe's poem sought ways to expand his knowledge, he ignored mathematics altogether. However, it is not true that mathematicians have been entirely neglected because the astronomer Johannes Kepler is the hero of Paul Hindemith's largely factual opera *Die Harmonie der Welt* which is better known by the masterly symphony based on it. Also, the heroine of George Bernard Shaw's play *Mrs Warren's Profession* is a mathematician and so gains independence from her mother's immoral wealth by her mathematical work. However, the play contains no general comments about mathematicians.

Our failure to learn anything from literary depiction leads us to deploy the sociologist's most powerful method, the social survey. This is the process whereby the random comments of a selection of people are transmuted by statistical sleight of hand into the hard facts of sociological science. Any survey of mathematicians must take into account that they are necessarily intelligent, so they must be compared with a suitably intelligent control group. A survey has been made which compares the academics of different subject groups. Although the details of this survey are not to hand, it is understood that one conclusion that it indicated asserted that the mathematicians were the most successful in marriage whereas the social scientists had the least success. We may ascribe this observation to the fact that many people study sociology because they are unsure of their social relationships, while mathematics demands qualities which are also beneficial in marriage. However, this poll suggests a problem. What would happen if a mathematician married a sociologist? In the only case that has been reported, the mathematician persuaded the sociologist to change to the humanities, a group of academics with a better marital record.

In the absence of further relevant surveys, we now proceed to use direct observation; that is, we deduce general principles from carefully selected anecdotes.

First we note that a remarkably high proportion of mathematicians are appreciators of some form of music, and a large minority are performers as well, some of them being musicians of professional standard. From his use of mathematics in his book *Les Fondements de la Musique dans la Conscience Humaine* it is clear that the celebrated conductor Ernest Ansermet was a mathematician, but the book is an efflorescence of existential philosophy which eschews autobiographical detail. It is widely believed that the affinity of mathematicians for music is due to parallels of form, but the variety of the styles of music involved militates against this view. A more attractive alternative theory is that mathematics and music balance in the mind, contrasting meaning without intrinsic emotion with emotion without intrinsic meaning.

If reputation is believed, mathematicians commonly take part in active sports, probably because concentration on mathematics is unusually tiring. For example, G.H. Hardy worked for only four hours per day, then played real tennis. Certainly mathematicians need to choose between short, concentrated bursts of work and longer, more inefficient, sessions - yet some major mathematicians seem capable of long, concentrated bursts of work. Considering how many mathematicians are sporty, very few have entered top-class sport, though some (such as Virginia Wade) have reached the highest levels.

It is easier to find examples of mathematicians who are adept at intellectual games. For example, John von Neumann even converted his experiences at the poker table into a new branch of mathematics, games theory. Nevertheless, the prize for a mathematician and games player must surely be awarded to Emanuel Lasker, who was World Chess Champion from 1894 to 1921.

As we have now exhausted the results of snooping in the corridors of mathematics, we will try the theoretical approach, that is, we will deduce the character of mathematicians from the nature of mathematical work.

Compared with work in other subjects, research work in mathematics is difficult, yet it is judged by very high standards of originality. Further, the certainty of mathematical theorems is offset by the fact that they are based on unproven axioms, so they are absolutely empty. However, the clear objectives make mathematics an easy subject to teach for anyone who has mastered it, provided that the following is always observed.

The golden rule

The teacher should inspire sometimes,
Enlighten often
And encourage always.

In order to obey this rule the teacher needs to bear in mind that a proposition that is to be taught must be true, even if it is not entirely clear, but that a clearer, restricted result is better than a more obscure, general result.

Naturally, the constant handling of the small change of logic affects the way mathematicians think, and therefore what they are. For example, the criticism of logic in manuscripts they are marking leads mathematicians to search for *non-sequiturs* in what they read, for otherwise the principles of logic would be used for incorrect purposes. Among the correct methods of argument there is mathematical induction, which was discovered by some unknown mathematician and since then every mathematician has taught it to his pupils, so that now every mathematician uses mathematical induction. Also, mathematicians frequently disprove results by means of counter-examples, although the habit is not confined to mathematicians because it was used by Aristotle.

The friends of mathematicians cannot always understand them. To prove this, let us assume that the statement is false, hence that mathematicians are always understood by their friends. This implies that all the people in a certain random sample of the population understand the principles of logic which are commonly used by mathematicians, such as *reductio ad absurdum*. But mathematics students, who are selected for their potential ability to use these principles, still find them difficult, so a random sample of people could not understand them. This contradiction proves that mathematicians are greatly misunderstood.

The practice of mathematics over a long period has some deeper effects, including the development of some virtues. Patience is developed by the need to invent an indirect strategy to reach an objective, to carry it out and then to accept the disappointment when the strategy fails, though the jubilation is correspondingly great if it succeeds. Modesty is developed by assessing the value of the results that can be obtained, because these are necessarily limited by the weakness of mathematical techniques. Newton expressed this idea in these words:

> "I do not know what I may appear to the world, but to myself I seem to have been only a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

A related feeling even causes some people to give up mathematics: that most mathematical problems are too unimportant to occupy a large part of life, and all the others are much too difficult.

Warning: Mathematics can damage your health.

Think about the health risks before doing mathematics.

First, the isolation of mathematics from other subjects makes it difficult to employ non-mathematical talents, with resulting frustration and unhappiness, or the diversion of much energy into pursuits outside mathematical work.

Secondly, mathematicians, like most intellectuals, are susceptible to the "fallacy of categories". Perhaps this is because mathematics does not employ value judgments, so it is not an art. Furthermore, it does not employ the empirical method, so mathematics is not a science. Consequently, mathematics does not exist. Or, perhaps, we have failed to prove that the arts and the sciences include all intellectual disciplines. The logical sciences, in which the method is deduction from axioms, seem to be an obstruction in the way of such a proof.

Thirdly, the increasing maturity of a mathematician's thought causes him to lose sight of the difficulties of those without his own experience. For example, the mathematician may see that a problem about numbers becomes easy if it is regarded as a problem about functions of numbers instead of numbers themselves, although he may fail to realise that the solution is incomprehensible to anyone who is not already familiar with functions.

Fourthly, the pursuit of mathematics leads to an overuse of logical analysis. For example, John von Neumann feared that global war was inevitable. By Murphy's law, sooner or later, there would be a dispute between a Warsaw Pact country and a NATO country, and this would lead to war between them. The treaties would bring the full power of both alliances into the war, then one side would use nuclear weapons to equal the other's strength, an exchange of the most powerful weapons would follow and the world would end. This analysis seems sound, but the global war has not started in 30 years. Presumably, we have missed some other factors, such as diplomacy and the effect of strategic analysis. Edward de Bono has introduced the concept of lateral thinking as a means of combating the excessive use of analysis.

Fifthly, mathematics induces the intellectual vice of "hardmindedness", the view that if an argument is complete then the proposition should be accepted.

The mathematician should always be aware that one of the following may be true:

(i) There may be other evidence

(ii) The opponent may regard the proposition as an axiom (or its denial) and all argument about it as irrelevant

(iii) The opponent may have used a proposition in his argument which he did not believe, so he is not concerned that it has been disproved

(iv) The opponent may need time to adjust to the conclusion

But, perhaps, despite all these efforts, Augustus de Morgan's words will remain true:

"It is easier to square the circle than to get round a mathematician."

Our investigations have shown that mathematicians do work that is hard, probably unsuccessful, performed in isolation, of negligible social prestige, unlikely to attract the opposite sex, poorly paid and not applicable to the central concerns of society. Surely, anyone who takes up such work must be eccentric? Indeed, as mathematical work is not central in society, mathematicians are eccentric for the best of all reasons: by definition.

To compensate for the indifference of society to them, mathematicians can at least ameliorate life for each other by swearing the following oath.

The Archimedean oath

The mathematician shall:

1. Never teach anything that does not have a precise proof.

2. Never publish any statement of which there is no written, detailed proof.

3. Never overfill the bath.

Also, mathematicians can console themselves by reflecting that they hold the key to a large part of human knowledge, although this often leads them to think that they actually possess all that knowledge. This can be true to such an extent that they even think that they understand mathematicians.

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MORE PYTHAGOREAN SNIPPETS

Are there Pythagorean triangles whose hypotenuse is a perfect square and with legs such that their difference is also a square? Yes. The smallest such triangle is 119, 120, 169.

Is there a Pythagorean triangle with legs of the same digits but in reverse order? Yes. 88,209, 90,288, 126,225.

No isosceles right triangle can be Pythagorean (its hypotenuse is incommensurable with a leg) but one can get as close to isosceles as one pleases. e.g. 21,669,693,148,613,788,330,547,979,729,286,307,164,015,202,..,768,699,465,346,081,691,992,338,845,992,696. The other leg is that number plus 1.

Are there Pythagorean triangles that have the same area? Yes. So far three. 1,380, 19,019, 19,069; 3,059, 8,580, 9,109; 4,485, 5,852, 7,373. Their common area is 13,123,110.

Submitted by Garnet J. Greenbury.

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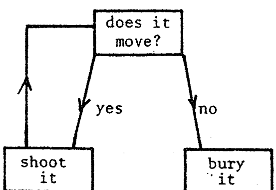
THE COMPUTERIZED ARMY:
MARK III



From Ian Stewart and John Jaworski: *Seven Years of Manifold.*

A PROBLEM FROM THE MOSCOW PAPYRUS

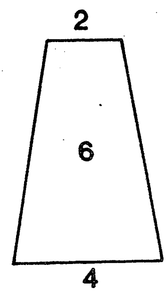
G.C. Smith, Monash University

The Moscow mathematical papyrus was discovered in the 1890s in the Necropolis of Dra Abu'l Negga in Egypt. It was purchased by W. Golenischev and after his death his collection, including this mathematical papyrus, was acquired by the Moscow Museum of Fine Arts. The papyrus was written in the later part of the Middle Kingdom (2160 BC - 1670 BC). It is more than 5 m long but only 8 cm in height. The manuscript was probably written about 1800 BC, but the ideas it contains are certainly much older - most of the surviving Middle Kingdom papyri containing mathematics are considered to be copies of (or to be based on) earlier ones which originated in the Old Kingdom period - perhaps as early as 2700 BC.

Here is a literal translation of one of the problems and its solution together with the figure which accompanies it:

"If you are told: a truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top; you are to square this 4; result 16. Your are to double 4; result 8. You are to square this 2, result 4. You are to add the 16 and the 8 and the four; result 28. You are to take 1/3 of 6; result 2. You are to take 28 twice; result 56. See, it is of 56. You will find it right."

This is no further explanation.



Problem.

Find what it is that the Egyptian scribe who wrote this papyrus was working out.

Answer on p.29.

LETTERS TO THE EDITOR

Further to the article on Yan-a-Bumfit (see *Function*, Vol.8, Parts 1, 2) this passage from Amy Stewart Fraser's volume of Edwardian recollections, *Roses in December*, may be of interest,

"Elizabeth Dodd recently described the Falkirk Tryst in her grandfather's day: how he travelled from Penrich in Cumbria to Falkirk to buy sheep, and how he, with the help of hired drovers, brought them south by easy stages .... Every morning the sheep were counted, to every man his drove, then on their way again. The counting was done by the old Celtic method -

Yan tan tethera methera pimp;
Sethera lethera hovera dovera dick;
Yanadick tanadick tetheradick metheradick bumfit;
Yanabumfit tanabumfit tetherabumfit
metherambumfit giggot.

Grandfather counted aloud, and a man stood by his side with a stick in which he made a notch every time Grandfather said Giggot."

This passage occurs in reference to nine droves each of two-hundred sheep.

Neil Cameron
Monash University.

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SOME CLAIMS TO CHECK

I have been looking for numbers whose squares end in 987654321.

I found these numbers:

111 111 111
888 888 889
880 642 361 .

There are many more such numbers. Is there a general formula?

D.R. Kaprekar
311 Devlali Camp, India.

[*We haven't checked these results, nor do we know of a general formula, but if the claims are true one can clearly generate infinitely more examples by adding numbers ending in a suitably large number of zeros to the examples given. Eds.*]

PROBLEM SECTION

We have had a few responses to our earlier problems, but think that perhaps more time is needed and so do not print solutions at this stage. Here, however, are some new problems.

PROBLEM 8.4.1. (Submitted by Isaac Nativ.)

Show that:

$$-i = \cfrac{1}{2i + \cfrac{1}{2i + \cfrac{1}{2i + \cfrac{1}{2i + \cfrac{1}{\ddots}}}}}$$

where $i^2 = -1$.

PROBLEM 8.4.2. (Due to Lewis Carroll, submitted by S.J. Newton.)

A room has a light switch at each corner. It is not possible by examining the switch to tell if it is on or off. The light will, however, be off unless all switches are in the "on" position. A person comes into the room and finds the light off, then presses each switch in turn with no result. He then presses again in order the first, second and third switches, still with no result. How should he proceed if he is to turn the light on?

PROBLEM 8.4.3. (Submitted by D.R. Kaprekar.)

If all the diagonals of a regular octagon are drawn, how many points of intersection are there?

PROBLEM 8.4.4. (Submitted by Garnet J. Greenbury.)

a, b, c are integers satisying $a^2 + b^2 = c^2$. Prove:

1. Of these integers, one is divisible by 3, another (possibly the same one) by 4, and one by 5;

2. The product abc is divisible by 60;

3. $ab/2$ (the area of a corresponding right-angled triangle) is never a perfect square;

4. The radius of the in-circle of the right-angled triangle is always integral.

PERDIX

The 1984 International Olympiad was held in Prague from July 2 to July 10. The Australian team performed very well when it is considered that this country is still new to such competitions. We won one silver and two bronze medals, and were perhaps a little unlucky that this was not two silver and two bronze. The results were:

Michael Peake	33/42	Silver
Alan Blair	25/42	Bronze
(missed by 1 mark on a Silver)		
Matthew Hardman	21/42	Bronze
Jonathan Ennes	16/42	
(missed by 1 mark on a Bronze)		
Andrew Jenkins	6/42	
John Kramer	2/42	
Total	103	

Australia was placed 15th overall, four places better than last year, our previous best. This is also the first time our total has exceeded 100 points.

* * * * * *

Solving problems

Seeing how to solve a problem in a competition is perhaps the greater part of the battle, but it is not the whole battle. You have to explain your solution; and in the Olympiad competitions this means a written explanation, presented preferably with impeccable logic and with clarity of argument and expression.

Learning how to write clear and convincing arguments giving solutions to problems is an important part of preparing for Olympiad and other competitions. Do not neglect it. A beautifully clear exposition indicates a total understanding of a problem. The effort to organise your written solution so that your argument has simple clarity helps to ensure that you have completely understood the problem and provides perhaps the best safeguard against having made any mistakes. Practice your exposition. If you are not completely happy with your written version of a solution, then try to polish it, write it again. Then try again if you think this is necessary. This will not be wasted time. It will help you to think your way more clearly through future problems.

In this issue I propose to look first at problem 2 that I left you with in the last issue of *Function* (Volume 8, part 3, inside back cover).

PROBLEM 2. (Sixth International Olympiad, 1964, problem 1)

(a) *Find all positive integers* n *for which* $2^n - 1$ *is divisible by 7.*

(b) *Prove that there is no positive integer* n *for which* $2^n + 1$ *is divisible by 7.*

Solution.

(a) The number $2^n - 1$ is divisible by 7 if $2^n = 7k + 1$, for some integer k, i.e. if the remainder on dividing 2^n by 7 is 1. Let us experiment with a few values of n:

n	2^n	remainder on division by 7
1	2	2
2	4	4
3	8	1
4	16	2
5	32	4
6	64	1
7	128	2
8	256	4
9	512	1

It seems clear that the answer to the problem is "all positive integers $3m$". This is our guess based on experiment. In fact this answer is correct. How do we show this?

The hint that was given in the last issue will help. You were there told that, if, for integers m, k and r,

$$m = 7k + r ,$$

then, for any integer s, ms and rs give the same remainder on division by 7. This is surely clear, because

$$ms = 7ks + rs :$$

the $7ks$ makes no difference to the remainder we get on division by 7.

So, to find the remainder on division of 2^n by 7, we merely need to work with the successive remainders:

	2^3	gives remainder 1
so	2^4	gives the same remainder as 1×2, viz. 2
	2^5	. . . . . 2×2, viz. 4
	2^6	. . . . . 4×2, viz. 1;

and since we are now back to our starting point with remainder 1, the cycle repeats itself: we get remainder 1 if and only if n is a multiple of 3.

(b) Similar arguments work for this part of the problem. The number $2^n + 1$ is divisible by 7 when $2^n + 1 = 7k$, for some integer k, i.e. when $2^n = 7k - 1$, i.e. when

$$2^n = 7(k-1) + 6 .$$

Thus $2^n + 1$ is divisible by n if 2^n gives remainder 6 on division by 7. But we saw in part (a) that the only remainders possible on division of a power 2^n by 7 are 1, 2 and 4. Hence for no positive integers n is $2^n + 1$ divisible by 7.

This completes the solution to problem 2.

Some general remarks

The kind of arguments used in solving problems 1 and 2 are those of what is called *modular arithmetic*. Modular arithmetic is concerned with the properties of the remainders you get when you divide one number by another. To be more precise, let us define the *residue* of the number n when it is divided by m (all numbers in modular arithmetic are integers) to be the number r for which there is an integer k such that

$$n = km + r ,$$

and $0 \leqslant r < m - 1$.

For example, the residue of 64 when divided by 7 is 1; the residue of 24 when divided by 11 is 2.

In general if two numbers n_1 and n_2, say, give the same residue when divided by m, we say that n_1 *is congruent to* n_2 *modulo* m . In symbols, and as a shorthand, we write this as

$$n_1 \equiv n_2 \pmod{m}. \qquad (*)$$

Thus, in particular, if the residue of n when divided by m is r we have

$$n \equiv r \pmod{m} ,$$

i.e. any integer n is congruent to its residue modulo m. In general (*) holds if and only if $n_1 - n_2$ is divisible by m.

The relation of congruence modulo m respects addition and multiplication (as stated in the next two results).

RESULT 1. *Let* $n_1 \equiv n_2 \pmod{m}$ *and* $n_3 \equiv n_4 \pmod{m}$. *Then* $n_1 + n_3 \equiv n_2 + n_4 \pmod{m}$.

Proof. $n_1 \equiv n_2 \pmod{m}$ means that $n_1 - n_2$ is divisible by m, $n_1 - n_2 = k_1 m$, say, for some integer m_1 . Similarly there exists an integer k_2 such that $n_3 - n_4 = k_2 m$. Hence, adding,

$$(n_1 + n_3) - (n_2 + n_4) = (k_1 + k_2)m ,$$

so that $n_1 + n_3 \equiv n_2 + n_4 \pmod{m}$.

RESULT 2. *Let* $n_1 \equiv n_2 \pmod{m}$ *and* $n_3 \equiv n_4 \pmod{m}$. *Then* $n_1n_3 \equiv n_2n_4 \pmod{m}$.

Proof. Let k_1 and k_2 be such that $n_1 - n_2 = k_1m$ and $n_3 - n_4 = k_2m$. Then

$$n_1n_3 - n_2n_3 = k_1mn_3 \tag{1}$$

and

$$n_2n_3 - n_2n_4 = k_2mn_2 , \tag{2}$$

multiplying by n_3 and n_2, respectively. Now add (1) and (2) to get

$$n_1n_3 - n_2n_4 = (k_1n_3 + k_2n_2)m$$

so that

$$n_1n_3 \equiv n_2n_4 \pmod{m} .$$

You will recognise that this so-called *modular arithmetic*, adding and multiplying modulo m, was what was involved in the solutions of problems 1 and 2.

We shall use results 1 and 2 again in the future.

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MORE ON PICK'S THEOREM

Function, Volume 8, Part 1, p.4 carried an article on Pick's Theorem, a result which enables us to compute the area of a polygon drawn such that its vertices lie on the points of a square lattice. In a recent issue (March, 1984) of *The Australian Mathematics Teacher*, which *your* mathematics teacher is very likely to subscribe to, a somewhat different proof is given, together with a number of references.

The author is Kevin Green, of Sydney CAE. He references Pick's original article which appeared in what seems to have been a rather obscure journal in Prague, 1899. Other accounts, which are more accessible, occur in:

H.S.M. Coxeter: *Introduction to Geometry*, (Wiley, 1969),
S.K. Stein: *Mathematics, The Man-Made Universe*, (Freeman, 1969),

both of which should be in your school or municipal library.

Some years ago it was fashionable in schools to use the teaching aids known as geoboards. These are merely square pieces of chipboard with nails hammered part way in at the vertices of a square lattice drawn on the board. Polygons may then be constructed by placing rubber bands over the nails. This saves a lot of drawing and allows for exploration of Pick's Theorem (and, of course, other matters).

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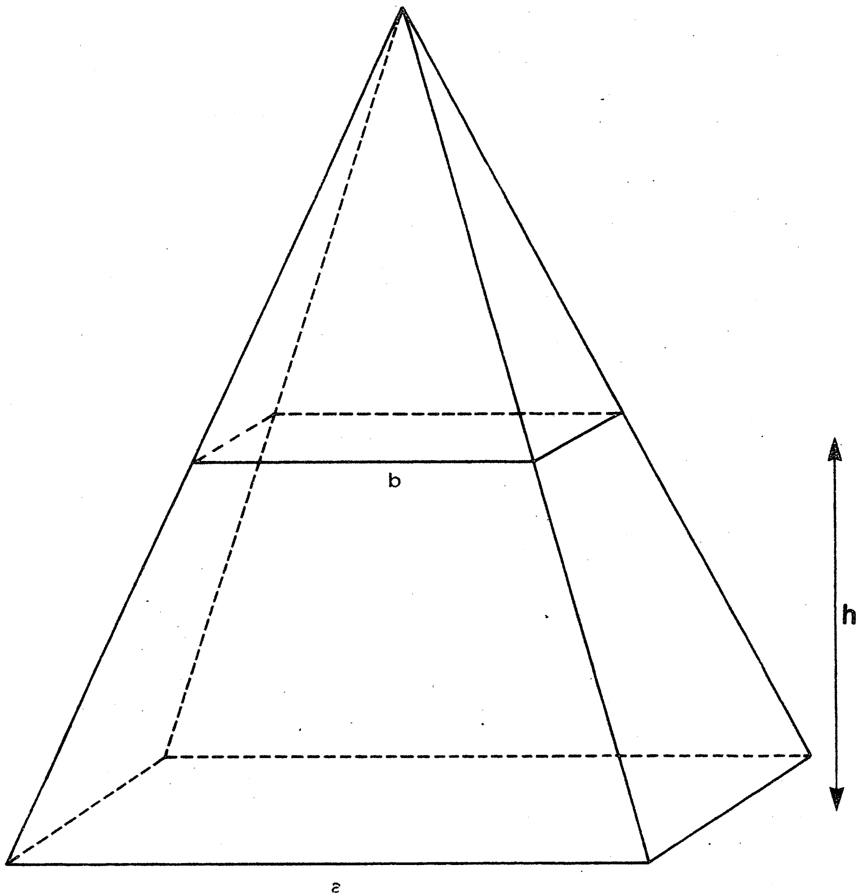
Solution. (See p.22.)

He is finding the volume of a truncated pyramid of square base; we would express this as

$$v = (a^2 + ab + b^2)h/3$$

where a, b are the sides of the squares and h is the vertical height of the truncated pyramid. In the Moscow papyrus $a = 4$, $b = 2$ and $h = 6$.



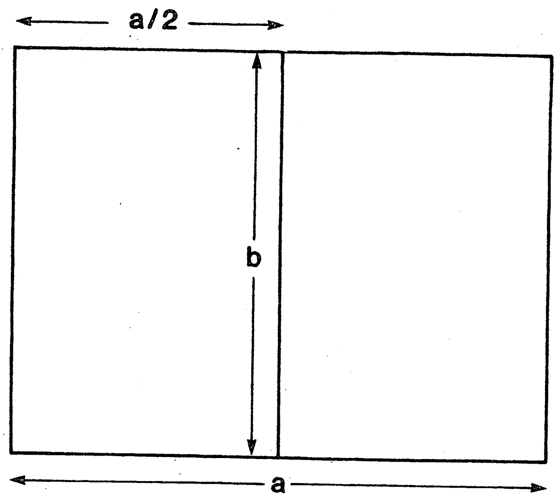
PAPERFOLDING

When metric standards were introduced into Australia, various other things changed as well. Most notably, quarto sheets of paper were replaced by a new size and shape known as A4. An A4 page is 297 mm long by 210 mm wide. *Function* is printed on A4 paper which is then folded and trimmed. But why are these dimensions chosen?

Often the answers to such questions rest on relatively arbitrary historical circumstance, but in this instance the reason is mathematical.

If we take the product of the two numbers quoted, we find an area close to 1/16 of a square metre. 16, the denominator, is 2^4 and this is the origin of the name A4. The A4 sheet is the result of folding a large sheet, of area almost exactly one square metre, in half four times. This produces, if the paper is now cut along the folds, 16 sheets of A4.

But this does not yet give us enough information. For area depends on both length and width. What we need is further information on, not merely the size of the sheet of paper, but also its shape.



Look at the diagram. If we fold a sheet of paper of length a and width b about the middle of its length (we assume $a > b$) we get a folded sheet of length b and width $a/2$ (subject to suitable restrictions - what are they?).

Suppose that we now impose the condition that *the new shape be the same as the old*. We then have that the ratio of $a/2$ to b must be the same as that of b to a. That is to say

$$\frac{a}{2b} = \frac{b}{a}$$

or

$$\left(\frac{a}{b}\right)^2 = 2$$

or

$$a = b\sqrt{2}\ .$$

This is exactly what has happened with the A4 paper. $210\sqrt{2}$ is very slightly less than 297 , but 297, 210 are the best integer approximations that will

(a) give a ratio close to $\sqrt{2}$

(b) give a product close to $10^6/16$.

The paper size produced by folding A4 in half is termed A5, and A4 is produced by folding A3, itself produced by folding A2, etc. At each stage a slight complication is produced by the need to specify the dimensions in an exact number of millimetres, but a very exact correspondence is now set up.

Another, less well-known series of paper sizes results from folding and cutting a larger sheet: 1 m by 1·414 m. This yields the B series, so that B5 paper measures 250 mm × 176 mm.

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THE CMU PROJECT

Several US colleges and universities have developed long-term schemes for computer learning. The Society for Industrial and Applied Mathematics (SIAM) report on the most ambitious of these, a cooperative venture between IBM and Carnegie-Mellon University (Pittsburgh). The following excerpts from *SIAM News* (July 1984) give an idea of the scope of the project.

On October 20, 1982, Carnegie-Mellon University (CMU) and International Business Machines (IBM) signed a three-year agreement to develop a prototype personal computing network. The plan calls for several thousand personal computer workstations, each between 20 and 100 times more powerful than current home computers, to be in place by 1986 and for seven thousand to be in place by 1990. The IBM agreement is part of a larger coordinated effort at CMU to integrate computing fully into the undergraduate and graduate curricula.

Personal computing at CMU will develop in two stages. Phase I, a two-year transition currently in progress, introduces personal computer workstations. Phase II will introduce the advanced personal computing environment now under development.

During Phase I, CMU will install approximately one thousand personal computers. Some are to be placed in clusters around the campus, while others sold to students and staff will be in dormitory rooms and offices.

The CMU distributed computing system will have several features lacking in current timesharing systems. As with any personal computers, access to and performance of the workstations on the network will not be affected by other users on the system. Unlike ordinary personal computers, however, shared central storage facilities will provide users access to their files from anywhere on the network. The computing power of the individual workstations will allow for customization by each user, making it easy for the user to perform desired tasks without comprehensive knowledge of the entire system.

The workstations will be able to maintain several contexts simultaneously, allowing users to move among tasks with no evidence of interruption and making it possible to reenter a task at a different workstation. The network will enable communication among all workstations on the network, providing capabilities for electronic mail, data and program sharing, and document transfers similar to current timesharing systems. Through connections to national networks users will also have access to facilities at other locations and institutions.

The system will eventually receive, generate and store video information in addition to traditional text files. The system configuration will allow smooth growth from a system with tens of workstations to a system with thousands of them. The network will be extremely reliable; when one personal computer malfunctions the network will continue to run so that the user can move to another workstation.

The system's cost is another advantage. Relative to capacity the price of personal computers is declining more rapidly than the price of larger scale systems. The network approach will thus provide superior computing facilities to the greatest number of users at the lowest cost.

The planned network will consist of four elements: individual personal computer workstations; clusters of workstations and various specialized facilities; a communications network; and a central computer facility.

The Vast Integrated Communication Environment (VICE) will resemble a giant time-sharing file system and is planned to be expandable and adaptable to changes in technology and needs.

Each personal computer workstation will consist of a 32-bit processor capable of one million instructions per second execution speed, one million bytes of random access memory, a virtual memory operating system, high-resolution bitmap graphics (with colour as an option), graphical input, audio output, video display, and keyboard. Local disk storage will be available as an option.

Clusters of 20 to 50 workstations, joined to file servers by a high speed local area network, will be installed at various locations to provide access to laser printers, input scanners, magnetic tape drives, floppy disk drives, and plotters.

The local area networks will be linked together and to the university's mainframe computers through a very large capacity backbone network. The mainframe facilities will include large scale on-line and archival storage, large scale computation facilities and other specialized equipment too costly for general use.

An integrated set of computer programs will enable use of these facilities. This software, known as Virtue Is Reached Through UNIXtm and EMACS (VIRTUE) will provide system-wide access to files as well as shared access to network resources. System-wide access to files will remove current machine-specific constraints on such access. VIRTUE will include a mail system, bulletin boards, and teleconferencing facilities for network-wide communication, as well as an integrated text/graphics editor and a window/icon screen manager.

A large network of personal computer workstations will generate many new ideas for using the network system. The Center for the Design of Education Computing (CDEC) will facilitate the development of those ideas by providing the faculty with technical support for projects, maintaining a loose coordination among the various projects, and serving as a liaison with the ITC so that software development will remain compatible with the evolving operating system. CDEC has an initial budget in excess of $1 million supplied by grants from the Carnegie Corporation and the Sloan Foundation, to fund: faculty release time to work on projects; editorial services for programs; and support for documentation in the Communication Design Center.

CMU is part of a consortium of 16 universities working with IBM to stimulate the development of educational computing applications. The planned network also permits workstations from other manufacturers. Currently CMU has, either in place or planned, workstations from Apple (Lisa and Macintosh), Victor (90000), Hewlett Packard, and DEC (Professional 350).

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SOUNDS EASY!

Mathematics is the science in which one uses easy words for hard ideas.

E. Kasner