Mathematical inquiry – from a snack to a meal

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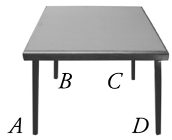
MathSnacks, by Marty Ross, Burkhardt Polster and QED (the cat) is the feature article of *Vinculum* that brightens up the back cover with its colourful presentation of several items (snacks) around a particular mathematical theme. These themes have covered for example *Pythagoras and Co.* ('Scary Scarecrow from the Wizard of Oz', 'Tantalising Triples', 'Perfect Proof' and 'Pizza Puzzle') the *Golden Ratio* ('Repeating Rectangle', 'Incredible Icosahedron', 'Fibonacci Formula', 'Ideal Irrational' and 'Ingenious Infinitum'), and, in the June 2005 edition of Vinculum *To Be or Not to Be – Four Variations on Mathematical Existence* ('Hairy Twin', 'Table Turning', 'Mathematical Meteorology' and 'Beautiful Points').

A key feature of ***MathSnacks*** is the presentation of the kernel of a mathematical argument or proof in a nutshell. From time to time it is worth exploring these in more detail than is available in such a concise format, and such an opportunity has arisen following correspondence from one of our readers, James Kershaw, in relation to the 'Table Turning' snack (shown in next column).

Table Turning



Put a square table on an irregular surface and chances are that it will wobble. However, by just turning it on the spot, you can always find a position in which all four legs touch the ground.

Proof: Suppose legs A, B, and C are touching the ground, and leg D is hovering in the air. So, if we anchor B and C, and force D to touch the ground, then A would be forced into the ground: bad for the table! Now rotate the table 90° clockwise, ensuring that A, B, and C are always touching the ground. Then, we again end up in a bad situation, as now D (which has assumed A's position) is poking into the ground. Since D starts out above the ground, and ends up below, there must be an intermediate position where D, and therefore all four legs, are touching the ground.

James inquired about the completeness of the proof …

"the problem of fitting coplanar points to an arbitrary surface, from memory, can only be generally solved for three points… that is, a three pointed stool can always be made stable and the fourth leg on a chair or table requires an extra degree of freedom to match".

Marty, Burkhardt and QED have advised that yes, there is an unstated assumption underpinning the argument – effectively that the ground didn't change level in a dramatic manner – and are happy to elaborate. Experience with tables and chairs at restaurants suggest that such an elaboration might be of some practical interest as well! They will provide a more detailed exposition of 'Table Turning' as a feature article in the forthcoming and final edition of Vinculum for 2005.

A central aspect of mathematics is its hypothetical-deductive nature, and proof, as a form of principled reasoning, is an essential part of working mathematically. But what does it mean to say that there is a proof of a proposition or conjecture? And how does one go about 'proving' something? In broad terms this means accepting certain things as given (axioms, structures, definition, assumptions and the like) and using certain agreed/accepted principles of reasoning (logical and mathematical) to derive

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certain other things (which mathematicians call theorems).

Proof requires a certain infrastructure, and has its own discourse, indeed there is an area of mathematics called *meta*-mathematics which studies the proof theory of mathematics (see, for example, Kleene, 1964).

There are various views on the nature of proof and its purposes, which in part also relate to philosophical considerations on the nature of mathematics and its objects, and how we work mathematically and come to know mathematical truths (see, for example, Jacquette, 2002). Proofs are supposed to be convincing, so, at a practical level, how does one come to be convinced of (believe in?) the efficacy of a particular proof, or the truth of what it is trying to show (see, for example, Lakatos, 1976)? Historical investigations show that many 'proofs' have required refinement, elaboration or correction as mathematical knowledge and understanding and standards of proof have developed over time. Kline (1980) comments:

> What then is mathematics if it is not a unique, rigorous, logical structure? It is a series of great intuitions, carefully sifted, refined and organized by the logic men are willing to apply at any time. The more they attempt to refine the concepts and systematize the deductive structures of mathematics, the more sophisticated are its intuitions. But mathematics rests upon certain intuitions that may be the product of what our sense organs, brains, and the external world are like. It is a human construction and any attempt to find an absolute basis for it is probably doomed to failure. Mathematics grows through a series of great intuitive advances, which are later established not in one step, but by a series of corrections of oversights and errors until the proof reaches the level of accepted proof for that time. No proof is final. The proofs are then revised and mistakenly considered proven for all time. But history tells us that this merely means that the time has not yet come for a critical examination of the proof (Kline, 1980, 313)

On the other hand, there is a very evident robustness in mathematics and its results, as evidenced by the strength of its applications, and the multiple approaches that can be taken to establish different proofs of the same propositions and conjectures, as in, for example, the third edition of *Proofs from THE BOOK* (2004) – a tribute to the spirit of Paul Erdös.

For some, the pleasure of constructing and/or discovering proofs is a priority, while for others inquiry into what 'proof' means is essential to understanding this vital aspect of mathematics. Readers who are interested in investigating practical *and* philosophical aspects of proof further may find the following references of interest.

References

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