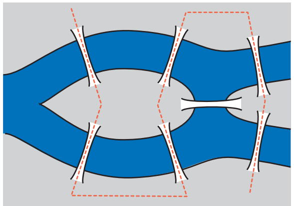
MathSnacks
Odder Even

by Marty Ross, Burkard Polster, and QED (the cat)

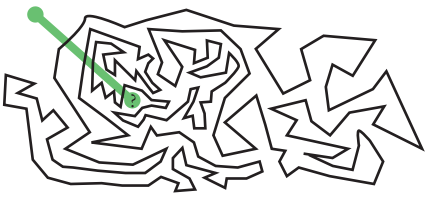
The Bridges of Königsberg



The river Pregel divides the city of Konigsberg into four regions connected by seven bridges. The great mathematician Leonhard Euler showed that it is impossible to go for a walk that takes you across every bridge exactly once.

Proof: If there was such a walk, then one of the four regions is neither its starting point nor its end point. So, whenever you enter this region via one bridge, you must leave it via a second. Hence there must be 2, 4, 6, ..., or other even number of bridges linking this region. Since every region is linked by an odd number of bridges, this is impossible, and, hence there is no such walk.

Amazing Maze



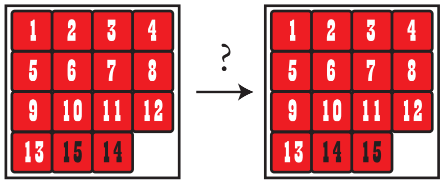
A closed curve without self-intersections divides the plane into an inside and an outside. If the curve is very convoluted, it is sometimes unclear whether a given point is inside or outside. To quickly determine the answer, walk from the given point until you are clearly outside, counting the number of times that you cross the curve. Every time you cross the curve you move from the inside to the outside, or vice versa. So, depending on whether this number is odd or even, the point is on the inside or the outside of the curve, respectively. In the diagram we cross four times. Consequently, the given point is on the outside.

Chocoholic Choice*

Starting with a rectangular bar of chocolate you and your chocolholic friend take turns breaking the bar along the lines that separate the squares. The person who makes the last break gets to eat all the chocolate. Who wins?

Answer: Every time you perform a break the number of pieces increases by one. This means that the total number of breaks is the number of squares in the bar minus one. So, if there are an even number of squares, the first person gets the chocolate, and otherwise the second person wins.

The 14-15 Puzzle*



The *14-15 Puzzle* was invented by Sam Loyd in 1878. He offered a prize of $1000 for solving the puzzle, that is for transforming the initial configuration on the left to that on the right by sliding the squares. Nobody ever claimed the prize! The reason, of course, is that the puzzle is impossible to solve.

Given any starting configuration, it is easy to determine whether it can be transformed to the final configuration on the right (though it is not easy to *prove* that the following test works). What you do is to cheat, and to "solve" the puzzle by repeatedly swapping the positions of any two tiles, counting the number of swaps it takes to do so. If this number is even, then the puzzle can be honestly solved, and is otherwise unsolvable. For example, in Sam Loyd's original puzzle, one swap suffices, and thus the puzzle cannot be solved.

Ripper References*

www.cut-the-knot.org
The Mathematical Puzzles of Sam Loyd, Edited by Martin